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CONTINENTAL O-1430-1 ENGINE

TORSIONAL VIBRATION CHARACTERISTICS AND CRITICAL SPEEDS OF THE CRANKSHAFT AND REDUCTION GEAR SYSTEM.

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TORSIONAL VIBRATION CHARACTERISTICS AND CRITICAL SPEEDS OF THE CRANKSHAFT AND REDUCTION GEAR SYSTEM

Object:- The purpose of this analysis was to predetermine the location of critical speeds, and to ascertain the vibration characteristics of a twelve cylinder flat engine, in order to intelligently develop its particular design for maximum reliable power output.

Summary:- The following pages contain in detail a special investigation, to determine how critical speeds originate, with special reference to the question, which of the many possible resonance speeds are really critical. By means of a working hypothesis described herein, the critical speeds for a twelve cylinder flat engine are calculated. No attempt is made to explain the theory of torsional vibration, nor the mathematical principles involved in the calculations, as the listed references clearly cover this matter.

The driving gas forces are taken from an actual indicator card, and, analytical considerations give the pressures due to mass forces. By the application of harmonic analysis to the gas and inertia forces, the harmonic components are determined. The normal elastic curve of the crankshaft system is calculated from its natural frequency, and the vector sum of the amplitudes for the normal elastic curve are evaluated by the aid of phase diagrams.

Curves showing the relative magnitude of the vibrations produced are plotted and form the basis for the conclusions given below.

Conclusions:- This analysis gives definite proof of the necessity of careful investigation in new design. Here we have a crankshaft system of comparative low frequency operating at high speeds, hence the possibility of encountering one of the harmonics of the lower order whose magnitude is great.

The basis of the conclusions in this analysis is the NATURAL FREQUENCY of the crankshaft system. It is quite evident that the magnitude of the third harmonic is so great, that it would be fatal to attempt to run the engine at this critical speed for any length of time.

In the calculation of the natural frequency of the crankshaft system (see Design Report #58) an assumption, based upon the best available experimental data, allowing for the looseness factor in the gear train, was made. This assumption, naturally, is open for criticism as it was based upon data from only two engines, the reduction gear of which, in neither case, is comparable to the type to be used in the Continental O-1430-1 engine. The reduction gear system in one case reduced the natural frequency 50%, while in the other, the natural frequency of the geared engine was 70% of that for the drive of the same design.

A glance at table 4, page 17, of this report shows the change in location of the critical speeds with a change in the natural frequency.

As the crankshaft itself has only 27.8 inches of equivalent length, in a system of 189 inches, it is evident that any change in the crankshaft would have little effect on the natural frequency of the system. It is, therefore, concluded to accept the crankshaft as designed. The quill shaft, however, having 34.3 equivalent length, lends itself to modification.

It is highly probable that, by the addition of the flexible quill shaft, to smooth out the torque variation to the propeller, we have reduced the natural frequency below its safe limits.

It is therefore concluded, that further studies be made on the two existing engines, from which experimental data on the effect of reduction gears was taken; to see if anything definite can be determined, to closer approximate the looseness factor for our reduction gearing. A further study will also be made using the Ranger, a 12-cylinder geared Vee engine with a quill drive to see what the looseness factor was in that case.

Further study should close up the gap in our present assumption of the looseness factor, and provision will be made in the present design, to give sufficient leeway in alteration of the quill shaft to escape dangerous critical speeds, if encountered, as predicted.

Procedure and Discussion:-

EXCITING TURNING FORCES OF GAS AND MASS PRESSURES:

Torsional crankshaft vibration is caused by the periodically changing torque moments, or torque variation, acting on the shaft. The moments originating in the engine have their source in the gas pressures, and the mass forces of the reciprocating parts. The mass forces of rotating parts, in so far as they are centrifugal forces, have no effect on torsional vibrations. If they are set up by tangential accelerations, they belong to the vibration appearances themselves, and do not appear as disturbances. In the following calculations we use the turning forces or torque, relative to the crank radius.

The first requisite in the study of the gas torsional forces is a knowledge of the indicator diagram, which can only be accurately determined by careful measurement. In the following computations, an especially fine indicator diagram was used. This diagram was taken with a Farnboro indicator equipped with a special valve, developed by the Research Department of the Continental Motors Corporation. A photographic copy of this diagram is shown on page of this report and curve C-26 shows the same diagram plotted on a P.V. scale.

At the time of recording the above card the engine was operating at the following conditions:

Hyper #2A Single Cylinder

Bore - - - - - - - - 5.500 In.   Stroke - - - - - - - - 5.000 In.

Connecting Rod Ratio $\frac{R}{L} = \lambda$ = .270

I.M.E.P. - - - - - - 206.4   Boost - - - - - - - - -18 In.Hg.

Intake Air Temp. ---------------- 222°F.

Jacket Coolant temp. in. ------- 224°F.

Jacket Coolant temp. out ------ 250°F.

The overturning force, or torque produced by the single cylinder is the tangential force at the crank radius due to both the inertia and gas forces. The gas pressures acting on the cylinder, along the cylinder axis, are obtained at various crank angles from the indicator diagram, while the inertia forces are calculated from the following equation:

$$F_i = -.0000284\, W_i\, RN^2 f_a$$

where

F_i = Inertia force, lbs.

W_i = Reciprocating weight = 6.383 lbs.

f_a = Crank angle factor for piston acceleration

$$f_a = \cos\theta \; \frac{R}{L} \cos 2\theta$$

where

θ = crank angle

R = Crank radius = 2.500 in.

L = Connecting rod length = 7.750 in.

The resultant force, acting in the direction of the cylinder axis, is obtained by adding algebraically, the inertia force and the gas force, at the crank angles considered. The torque caused by the forces acting in the cylinder at any instant during the cycle, is calculated from the equation:

$$T = F_a\, R\, f_v$$

where

T = Torque lbs. inches

F_a = Resultant force along cylinder axis, lbs.

R = Crank radius, inches

f_v = Crank angle factor for piston velocity

$$f_v = \sin\theta + \frac{1}{2}\frac{R}{L}\sin 2\theta$$

The tabulation of the above calculation and the selected values from the indicator diagram are given in table 1 on page 15.

Curve C 27 is the tangential force diagram plotted to a convenient scale showing its basic period of 2 revolutions.

This tangential force diagram is subjected to an harmonic analysis, the coefficients of the harmonics being determined mathematically. In order to avoid errors it is necessary to divide the curve into at least 48 ordinates, because the torque diagram for one cylinder has a long period (2 revolutions), while the actual torque curve for a twelve cylinder combination has a period of 1/6 revolution.

The periodically varying cylinder torque is made up of a large number of harmonics of varying amplitude and frequency. These harmonics may be evaluated by applying Fourier's theorem, which states, that any periodic curve may be resolved into a constant term; and a fundamental sine or cosine wave, together with harmonics of two, three, four, etc. times the frequency of the fundamental. Curve C28 shows the harmonic analysis of the turning effort curve of the engine under consideration. The fundamental, or first harmonic is designated as being of the one-half order since its frequency is one-half the engine speed, the second harmonic, order No. 1, etc.

When the crankshaft is revolving at such a speed that one of these harmonics synchronizes with the natural frequency, resonance occurs and the engine is said to be at a critical speed of that order.

For a detailed explanation of the method of determining the numerical value of the harmonic coefficients, see reference No. 7

$$a_k = \frac{2}{n} \sum y_r \cos k_r = \frac{2}{n} \left(y_o \cos k_{xo} + y_1 \cos k_{x1} + --- + y_{n-1} \cos k_{xn-1} \right)$$

$$b_k = \frac{2}{n} \sum y_r \sin kx_r = \frac{2}{n} \left(yo \sin kx_o + y_1 \sin kx_1 + --- y_{n-1} \sin kx_{n-1} \right)$$

$$Ck = \sqrt{a_k^2 + b_k^2}$$

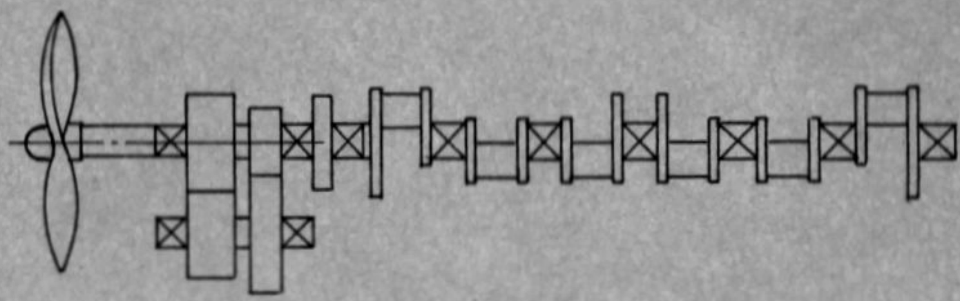
where k = Harmonic coefficient number.

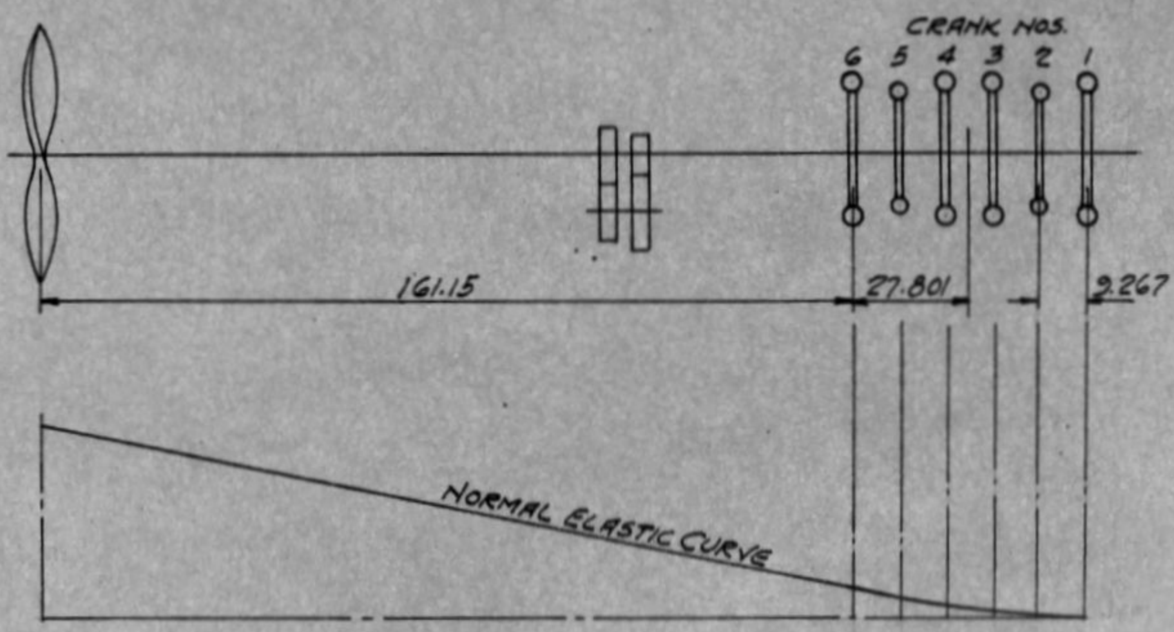
The results of the above analysis are given in table 3 and curve C29 in pounds per each square inch of piston area.

The fundamental period of the mass torque is one revolution in contrast to two in the gas torque, hence the effect of the mass torque on the harmonics of the combined gas and inertia force will only be evident in the even harmonics. As the higher harmonics do not appear in the mass torques, it follows that the harmonics of the combined gas and inertia forces of the higher order are not affected by the mass forces. It is evident that in a study of resonances of the higher order, it is not necessary to consider the inertia forces. However, in high speed engines with a reduction gear, the natural frequency of the system is reduced; and the lower harmonics become of great importance, as they are likely to appear in the operating range of the engine.

NORMAL ELASTIC CURVE OF CRANKSHAFT SYSTEM:

In order to consider the effect of the torque on the various cranks of the shaft, it is necessary to know its static deflection curve. This may be calculated from the natural frequency. (See reference No. 2).





The static deflection curve for a 12-cylinder, 7 bearing crankshaft system, such as the Continental 0-1430-1, may be calculated by assuming the respective masses as concentrated on each crank and calculating the necessary force to accelerate them.

In design, Report No. 58 (see reference No. 8) I_1, the moment of inertia of the crankshaft system, is given in lbs. in^2. The frequency of oscillation of a torsion pendulum is $f = \frac{1}{2\pi}\sqrt{\frac{C}{I}}$ in which C is the torque in lbs. ft. per radian, required to deflect the torsion member, and I is the moment of inertia of the pendulum in slug ft^2. It is necessary, therefore, to express I in slug ft^2. which is $\frac{I}{12 \times 32.2}$

$$I_1 = \frac{143.42}{12 \times 32.2} = .3715 \text{ (with counterweight)}$$

$$I_2 = \frac{120.91}{12 \times 32.2} = .320 \text{ (without counterweight)}$$

The frequency of free harmonic vibration is

$$f = \frac{p}{2\pi}$$

$$\therefore p^2 = (2\pi f)^2$$

$$f = 74.8$$

$$\therefore p^2 = 220,900$$

where K = torque moment necessary to produce an angle of twist of one radian in the shaft,

$$K = \frac{GI_p}{l}$$ for shaft cross-section

$$I_p = 2I = 2 \times 3.42 = 6.84$$

$$l = 9.26 \text{ (see sketch)}$$

$$\therefore K = \frac{11,500,000 \times 6.84}{9.26} = 8,500,000$$

Where λ_1 is the deflection produced in crank farthest from the propeller with a given torque moment, and $\lambda_2, \lambda_3, \lambda_4$ etc are the respective deflections of the other cranks,

$$\lambda_2 = \lambda_1 - \frac{I_1 p^2}{K_1} \lambda_1$$

Let λ_1 = 1.000 or 100% of the deflection

then $\lambda_2 = 1.000 - \frac{.3715 \times 220,900}{8.5 \times 10^6} = 1.000 - .00966$

λ_2 = .99034

$$\lambda_3 = \lambda_2 - \frac{p^2}{K_2} (I_1 \lambda_1 + I_2 \lambda_2)$$

$$\lambda_3 = .99034 - \frac{220,900}{8.5 \times 10^6} (1.000 \times .3715 \quad .320 \times .99034)$$

λ_3 = .99034 - .0228 x .6885 = .97519

$$\lambda_4 = \lambda_3 - \frac{p^2}{K_3} (I_1 \lambda_1 + I_2 \lambda_2 + I_3 \lambda_3)$$

= .97519 - .0228 (.3715 .317 .362) = 97519 - .024

λ_4 = .95119

λ_5 = 95119 - .0228 (1.0505 95119 x 3715) = 95119 - .032

λ_5 = .92919

$$\lambda_6 = \lambda_5 - \frac{p^2}{K_5} (I_1 \lambda_1 + I_2 \lambda_2 + I_3 \lambda_3 + I_4 \lambda_4 + I_5 \lambda_5)$$

= .92919 - .0228(1.403 92919 x .32) = .92919 - .038

λ_6 = .89049

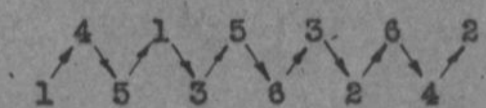
The various values used in the above calculations are given in Table #5 page No. 17 while Curve C30 shows the values of plotted against the equivalent length of the crankshaft. This curve represents the normal elastic curve of the system.

COMBINED EFFECT OF SEVERAL CYLINDERS--ORIGIN OF CRITICAL R.P.M.

In an engine composed of several cylinders, the individual harmonics of the different cylinders are superimposed, and their resultant partly neutralize, and partly augment each other. It is, however, to be noted that when certain harmonics for all cylinders together neutralize each other, they need not necessarily neutralize each other, in their effect as a vibration exciter, because the harmonic forces apply at different points on the system, where different vibratory deflections prevail.

Not all critical speeds are equally dangerous. The relative magnitude of the vibrations produced depends upon the value $D_k \Sigma \alpha_1 \sin \beta_1$ where D_k is the value of the turning effort harmonic, and $\Sigma \alpha_1 \sin \beta_1$ is the vector sum of the normal deflection obtained from the normal elastic curve at each cylinder.

On pages 18, 19, and 20 are plotted the vector diagrams for the harmonics of the twelve cylinder engine with the firing order of



and a firing interval of 60° of crank rotation.

The explanation of the diagram is as follows: For each order of critical speed, a phase diagram is constructed, 360° representing one complete vibration. Thus, for the one-half order critical speed (1st. harmonic), the vectors will be equally spaced around the circle as shown on page 18 for the 1st harmonics.

For the first order (second harmonic) critical speed, since one vibration is completed in half the time, the angles will be doubled as shown on page 18 for the 2nd. harmonic. We notice that only in the case of 12th. harmonic, addition to the intensity of 12 times the value for one cylinder takes place. It is also apparent that after the 12th harmonic the diagrams repeat.

Pages 18, 19, and 20 show the harmonics for the 12-cylinder engine in vector diagram, in their respective phases from 1st. to 12th. and for each harmonic the value of the expression is determined. Here the setting of the phases of the exciting torsion-forces to the natural vibration was selected, that

$\Sigma\alpha_1 \sin\beta_1$ becomes maximum, which denotes resonance.

The phase setting of the natural vibration is shown by the dotted arrow in the vector diagram, and was found by means of successive trials.

In curve C31 the computed values for $\Sigma\alpha_1 \sin\beta_1$ are plotted against the harmonics. This curve shows clearly the comparatively resonance deflections for the various harmonics by equally strong excitation and damping. We see that certain harmonics which neutralize each other in their resultants by no means neutralize each other in the amount of work. On the other hand, not all resonance cases can be considered as critical r.p.m.'s since their magnitude is slight.

In table 6, we give the computation of the values

$$D_k \Sigma\alpha_1 \sin\beta_1$$

which considers the different intensities of the excitation and, thereby, determines the danger of critical r.p.m.'s within the operating range of the engine.

Curve C 32 shows the values of $D_k \Sigma\alpha_1 \sin\beta_1$ plotted against the harmonics.

Curve C33 and table 7 shows the amount of work done by the harmonics of a 12-cylinder V engine taken from ref. No. 3. This table and curves were extended from the data contained within the reference article to include the lower harmonics; and is, in itself, a check upon the work contained in this report.

Curve C34 shows, on a larger scale, the computed value of $D_k \sum \alpha_1 \sin \beta_1$ for both a 12-cylinder flat engine and a 12-cylinder 60° V engine. However, these two curves are not comparable as to relative magnitude from one to another.

This preliminary report is a tabulation of the investigation carried on to date; and shows the necessity of further study in order to obtain a sufficiently close approximation of the natural frequency of the crankshaft, reduction gear, and propeller system.

TABLE NO. 1 SINGLE CYLINDER OVERTURNING FORCE

$$T = (F_g + F_i) \times R \times f_v$$

The values for the gas force used below were taken from Farnboro
Indicator Card No. 6-12-34 No. 9, 206.4 I.M.E.P.

(All values are lbs.per sq.in. piston area) $f_v = (\sin\theta + \frac{1}{2}\sin 2\theta)$

CRANK ANGLE	F_g GAS FORCE	F_i INERTIA FORCE	$F_a = F_g + F_i$	$F_a \times R$	f_v	$T = F_a R f_v$	CRANK ANGLE	F_g GAS FORCE	F_i INERTIA FORCE	F_a $F_g + F_i$	$F_a \times R$	f_v	$T = F_a R f_v$
0	6.75	-227.2	-220.45	-551	0	0	360	280	-227.2	52.80	-132	0	0
15	7.84	-214.2	-206.36	-516	.3394	-175	375	615	-214.2	400.8	1000	.3394	339
30	6.2	-176.7	-170.5	-426	.6395	-272	390	645	-176.7	468.3	1170	.6395	748
45	4.31	-121.5	-117.19	-293	.8684	-254.5	405	468	-121.5	346.50	866	.8684	752
60	2.7	- 58.3	- 55.6	-139	1.0055	-139.5	420	330	- 58.3	271.7	679	1.0055	682
75	1.57	3.47	5.04	12.6	1.0465	13.2	435	239	3.47	242.47	607	1.0465	635
90	1.5	55.4	56.90	142.2	1.0000	142.2	450	185	55.4	240.4	600	1.0000	600
105	1.57	92.4	93.97	235	.8853	208	465	150	92.4	242.4	607	.8853	537
120	2.43	113.6	116.03	290	.7265	211	480	127	113.6	240.6	600	.7265	436
135	3.5	121.4	124.90	312.5	.5458	170.5	495	109	121.4	230.4	576	.5458	315
150	5.12	121	126.12	315.2	.3605	113.5	510	92	121	213	532	.3605	192
165	6.8	118	124.80	312	.1782	55.4	525	75	118	193	482	.1782	86
180	8.35	116.6	124.95	312.5	0	0	540	51	116.6	167.6	419	0	0
195	10.2	118	128.20	320.5	-.1782	-57.1	555	32	118	150	375	-.1782	-66.8
210	11.9	121	132.9	332	-.3605	-119.7	570	25.5	121	146.5	366	-.3605	- 132
225	13.7	121.4	135.1	338	-.5458	-184.5	585	21.9	121.4	143.3	358	-.5458	-195.5
240	16.5	113.6	130.1	325	-.7265	-236	600	18.6	113.6	132.2	330.5	-.7265	- 240
255	20.0	92.4	112.40	281	-.8853	-248.5	615	15.1	92.4	107.5	269	-.8853	- 238
270	30	55.4	85.4	213.2	-1.0000	-213.2	630	11.6	55.4	67.	167.5	-1.0000	-167.5
285	40	3.47	43.47	108.6	-1.0465	-113.7	645	8.65	3.47	12.12	30.3	-1.0465	-31.7
300	60	- 58.3	1.7	- 4.25	-1.0055	4.27	660	6.95	-58.3	-51.35	-141	-1.0055	141.7
315	90	-121.5	-31.5	- 78.7	- .8684	68.4	675	5.4	-121.5	-116.1	-290	-.8684	252
330	140	-176.7	- 36.7	- 91.7	- .6395	58.6	690	4.6	-176.7	-172.1	-430	-.6395	274.8
345	200	-214.2	- 14.2	- 35.5	- .3394	12.05	705	4.32	-214.2	-209.88	-524	-.3394	177.6

TABLE NO. 2

ORDINATES FOR TANGENTIAL FORCE DIAGRAM C 27

n	T	n	T	n	T	n	T
1	0	13	0	25	0	37	0
2	-175	14	-57.1	26	339	38	-66.8
3	-272	15	-119.7	27	748	39	-132
4	-254.5	16	-184.5	28	752	40	-195.5
5	-139.5	17	-236	29	682	41	-240
6	13.2	18	-248.5	30	635	42	-238
7	142.2	19	-213.2	31	600	43	-167.5
8	208	20	-113.7	32	537	44	-31.7
9	211	21	4.27	33	436	45	141.7
10	170.5	22	68.4	34	315	46	252
11	113.5	23	58.6	35	192	47	274.8
12	55.4	24	12.05	36	86	48	177.6

TABLE NO. 3

HARMONICS OF GAS PRESSURES AND INERTIA FORCES

K	A_k	B_k	C_k	β_k	
1	-149.35	-116.57	189.4	232 deg.	2 min.
2	75.17	231.06	242.9	18 deg.	1 min.
3	1.56	-175.5	175.5	359 deg.	30 min.
4	29.05	- 85.05	89.8	341 deg.	9 min.
5	38.38	- 91.54	99.2	337 deg.	15 min.
6	- 39.77	- 39.45	56.0	225 deg.	14 min.
7	36.57	- 42.61	57.03	319 deg.	22 min.
8	- 36.08	16.53	39.7	114 deg.	37 min.
9	29.77	13.51	32.66	65 deg.	35 min.
10	- 23.95	7.24	25.05	106 deg.	49 min.
11	19.43	1.78	19.5	84 deg.	46 min.
12	- 15.22	- 1.68	15.3	263 deg.	42 min.
13	11.99	- 1.92	12.15	279 deg.	6 min.
14	- 9.02	- 5.77	10.70	237 deg.	24 min.
15	7.10	5.15	8.77	54 deg.	3 min.
16	- 5.94	- 6.00	8.44	224 deg.	43 min.
17	3.71	5.01	6.23	36 deg.	31 min.
18	- 3.34	- 4.19	5.36	218 deg.	34 min.
19	1.08	4.00	4.14	15 deg.	7 min.
20	- 1.64	2.68	3.14	148 deg.	33 min.
21	- 0.274	3.00	3.01	174 deg.	47 min.
22	0.94	1.38	1.67	34 deg.	16 min.
23	0.01	1.06	1.06	0 deg.	32 min.
24	1.17		1.17		

TABLE NO. 4

CRITICAL SPEEDS AT VARIOUS FREQUENCIES

Harmonic	Order of Critical Speed	F R E Q U E N C Y			
		40% Assumption f = 63.7	50% Assumption f = 74.8	70% Assumption f = 882	No Assumption f = 101
3 Rd.	1-1/2	2700	2992	3520	4050
4	2	1910	2240	2670	3040
5	2-1/2	1530	1795	2120	2420
7	3-1/2	1090	1270	1520	1730
9	4-1/2	850	1000	1175	1350
12	6	636	745	880	1010

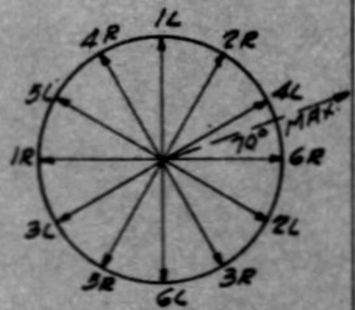
TABLE NO. 5

MOMENTS OF INERTIA AND PERCENTAGE DEFLECTION

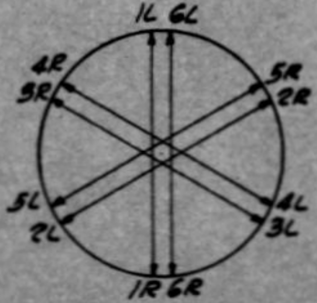
Mass No.	I Slug Ft.^{2}	% Deflection
1	.3715	1.000
2	.320	.990
3	.3715	.975
4	.3715	.951
5	.320	.929
6	.3715	.890

VECTOR DIAGRAMS

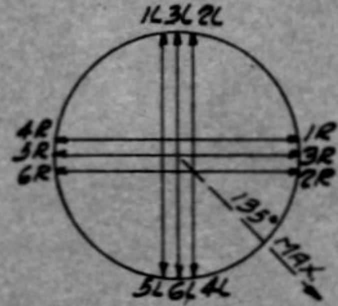
1ST HARMONIC



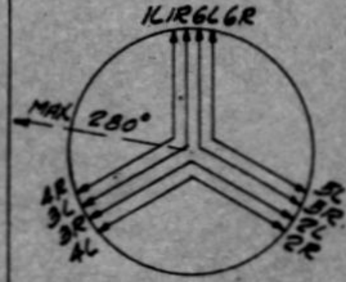
2ND HARMONIC



3RD HARMONIC



4TH HARMONIC

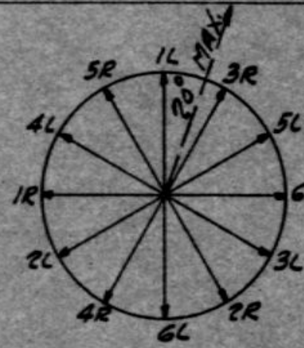


K		1ST HARMONIC			2ND HARMONIC			3RD HARMONIC			4TH HARMONIC		
CYL №	α_1	β_1	$\sin \beta_1$	$\alpha_1 \sin \beta_1$	β_1	$\sin \beta_1$	$\alpha_1 \sin \beta_1$	β_1	$\sin \beta_1$	$\alpha_1 \sin \beta_1$	β_1	$\sin \beta_1$	$\alpha_1 \sin \beta_1$
1-L	1.000	70°	.9397	.939	0°	0	0	135°	.7071	.707	280°	-.9848	-.985
2-L	.990	310°	-.7660	-.758	120°	.866	.858	135°	.7071	.700	160°	.3420	.338
3-L	.975	190°	-.1736	-.169	240°	-.866	-.845	135°	.7071	.690	40°	.6428	.627
4-L	.951	10°	.1736	.165	240°	-.866	-.824	315°	-.7071	-.672	40°	.6428	.611
5-L	.929	130°	.7660	.712	120°	.866	.805	315°	-.7071	-.657	160°	.3420	.318
6-L	.890	250°	-.9397	-.835	0°	0	0	315°	-.7071	-.629	280°	-.9848	-.876
1-R	1.000	160°	.3420	.342	180°	0	0	45°	.7071	.7071	280°	-.9848	-.985
2-R	.990	40°	.6428	.637	300°	-.866	-.858	45°	.7071	.700	160°	.3420	.338
3-R	.975	280°	-.9848	-.960	60°	.866	.845	45°	.7071	.690	40°	.6428	.627
4-R	.951	100°	.9848	.934	60°	.866	.824	225°	-.7071	-.672	40°	.6428	.611
5-R	.929	220°	-.6428	-.597	300°	-.866	-.805	225°	-.7071	-.657	160°	.3420	.318
6-R	.890	340°	-.3420	-.304	180°	0	0	225°	-.7071	-.629	280°	-.9848	-.876
$\Sigma \alpha_1 \sin \beta_1$				+ .105			0			+.278			+.066

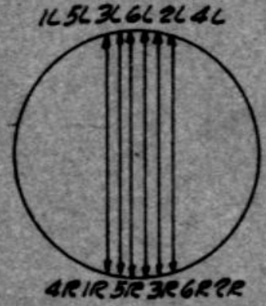
K	5TH HARMONIC	6TH HARMONIC	7TH HARMONIC	8TH HARMONIC

VECTOR DIAGRAM

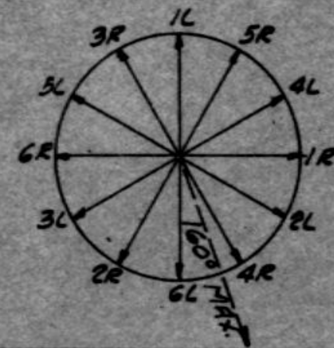
5TH HARMONIC



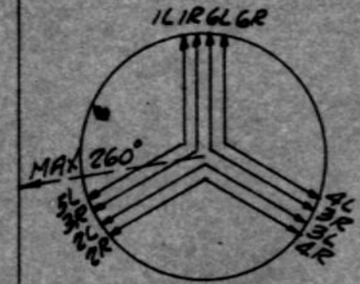
6TH HARMONIC



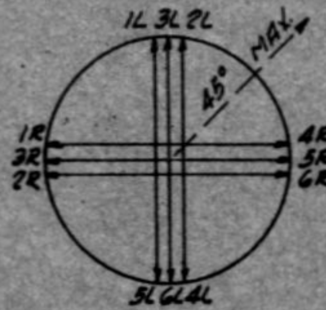
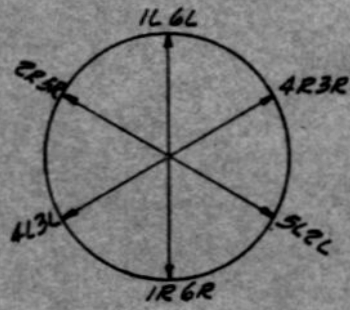
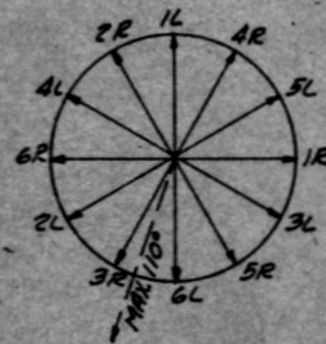
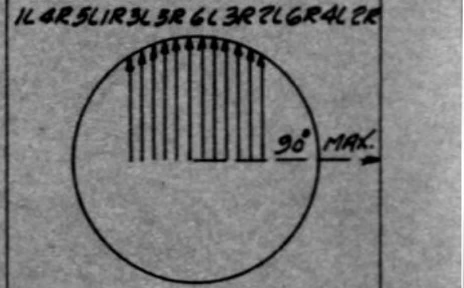
7TH HARMONIC



8TH HARMONIC



CYL. No	α_1	β_1	$\sin\beta_1$	$\alpha_1 \sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1 \sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1 \sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1 \sin\beta_1$
1-L	1.000	20°	.342	.342	0°	0	0	160°	.342	.342	260°	-.9848	-.9848
2-L	.990	140°	.6428	.636	0°	0	0	40°	.6428	.636	20°	.342	.3386
3-L	.975	260°	-.9848	-.960	0°	0	0	280°	-.9848	-.960	140°	.6428	.6267
4-L	.951	80°	.9848	.936	0°	0	0	100°	.9848	.936	140°	.6428	.6113
5-L	.929	320°	-.6428	-.598	0°	0	0	220°	-.6428	-.598	20°	.342	.3177
6-L	.890	200°	-.342	-.304	0°	0	0	340°	-.342	-.304	260°	-.9848	-.8765
1-R	1.000	110°	.9397	.939	180°	0	0	70°	.9397	.939	260°	-.9848	-.9848
2-R	.990	230°	-.766	-.758	180°	0	0	310°	-.766	-.758	20°	.342	.3386
3-R	.975	350°	-.1736	-.169	180°	0	0	190°	-.1736	-.169	140°	.6428	.6267
4-R	.951	170°	.1736	.165	180°	0	0	10°	.1736	.165	140°	.6428	.6113
5-R	.929	50°	.766	.712	180°	0	0	130°	.766	.712	20°	.342	.3177
6-R	.890	290°	-.9397	-.836	180°	0	0	250°	-.9397	-.836	260°	-.9848	-.8765
$\Sigma \alpha_1 \sin\beta_1$				+ .105			0			+.105			+.066

K	9TH HARMONIC	10TH HARMONIC	11TH HARMONIC	12TH HARMONIC
VECTOR DIAGRAM				

CYL NO	α_1	β_1	$\sin\beta_1$	$\alpha_1\sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1\sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1\sin\beta_1$	β_1	$\sin\beta_1$	$\alpha_1\sin\beta_1$
1-L	1.000	45°	.7071	.707	0	0	0	110°	.9396	.939	90°	1.000	1.000
2-L	.990	45°	.7071	.700	240°	-.866	-.858	230°	-.766	-.758	90°	1.000	.990
3-L	.975	45°	.7071	.690	120°	.866	.845	350°	-.1736	-.169	90°	1.000	.975
4-L	.951	225°	-.7071	-.672	120°	.866	.824	170°	.1736	.165	90°	1.000	.951
5-L	.929	225°	-.7071	-.657	240°	-.866	-.805	50°	.766	.712	90°	1.000	.929
6-L	.890	225°	-.7071	-.629	0	0	0	290°	-.9396	-.836	90°	1.000	.890
1-R	1.000	135°	.7071	.707	180°	0	0	20°	.342	.342	90°	1.000	1.000
2-R	.990	135°	.7071	.700	60°	.866	.858	140°	.6428	.636	90°	1.000	.990
3-R	.975	135°	.7071	.690	300°	-.866	-.845	260°	-.9848	-.960	90°	1.000	.975
4-R	.951	315°	-.7071	-.672	300°	-.866	-.824	80°	.9848	.936	90°	1.000	.951
5-R	.929	315°	-.7071	-.657	60°	.866	.805	320°	-.6428	-.598	90°	1.000	.929
6-R	.890	315°	-.7071	-.629	180°	0	0	200°	-.342	-.304	90°	1.000	.890
$\Sigma\alpha_1\sin\beta_1$				+.278			0			+.105			+11.47

D.R. #5560 24 pages
Page 20

Table No. 6
Computation of the amount of Work D_k

Harmonic Number K	Order of Critical	C_k	$D_k = \frac{C_k}{K^2}$	$\alpha_1 \sin \beta_1$	$D_k \alpha_1 \sin \beta_1$
1	1/2	189.4	189.4	.105	19.9
2	1	242.9	60.9	0	0
3	1 1/2	175.5	19.5	.278	5.42
4	2	89.8	5.56	.066	.367
5	2 1/2	99.2	3.96	.105	.416
6	3	56	1.55	0	0
7	3 1/2	57.03	1.16	.105	.122
8	4	39.7	.62	.066	.041
9	4 1/2	32.66	.395	.278	.011
10	5	25.05	.250	0	0
11	5 1/2	19.5	.161	.105	.0169
12	6	15.3	.106	11.47	1.22
13	6 1/2	12.15	.072	.105	.0073
14	7	10.70	.0546	0	0
15	7 1/2	8.77	.039	.278	.0111
16	8	8.44	.0329	.066	.00256
17	8 1/2	6.23	.0215	.105	.00226
18	9	5.36	.0165	0	0
19	9 1/2	4.14	.0115	.105	.00121
20	10	3.14	.0078	.066	.000512
21	10 1/2	3.01	.0068	.278	.00189
22	11	1.67	.0034	0	0
23	11 1/2	1.06	.0020	.105	.00021
24	12	1.17	.0020	11.47	.0229

TABLE No. 7

COMPUTATION OF THE AMOUNT OF WORK DONE BY THE HARMONICS OF A 12 CYLINDER 60° VEE ENGINE

Harmonic	Order of Critical Speed	E_k	$D_k = \frac{E_k}{k^2}$	$\Sigma\alpha_1 \sin\beta_1$	$D_k \Sigma\alpha_1 \sin\beta_1$
1	1/2	20.	2000.	0.70	1400.
2	1	20.	500.	0.33	165.
3	1-1/2	17.	183.	1.75	320.
4	2	3.90	24.4	0.18	4.4
5	2-1/2	10.	40.	0.88	35.2
6	3	2.55	7.10	0.00	0.00
7	3-1/2	6.20	12.60	0.88	11.10
8	4	3.90	6.10	0.18	1.10
9	4-1/2	3.70	4.57	1.75	8.00
10	5	2.90	2.90	0.33	0.95
11	5-1/2	2.30	1.90	0.70	1.33
12	6	1.90	1.32	8.58	11.30
13	6-1/2	1.60	0.95	0.70	0.67
14	7	1.30	0.66	0.33	0.22
15	7-1/2	1.05	0.47	1.75	0.82
16	8	0.90	0.35	0.18	0.06
17	8-1/2	0.70	0.24	0.88	0.21
18	9	0.50	0.15	0.00	0.00

Note:- Part of the above table is taken from reference no. 3 and the values for the first 6 harmonics calculated from the data contained within the reference article.

REFERENCES

1. "Vibration Characteristics of Aircraft Engine Crankshafts", A.C.I.C. Report No. 664 by Ford L. Prescott.

2. "Vibration Problems in Engineering" by S. Timoshenko, D. Van Nostrand Co.

3. "Torsional Vibration in Vertical Engines" by A. Stieglitz Luftfahrtforschung, July 24, 1929.

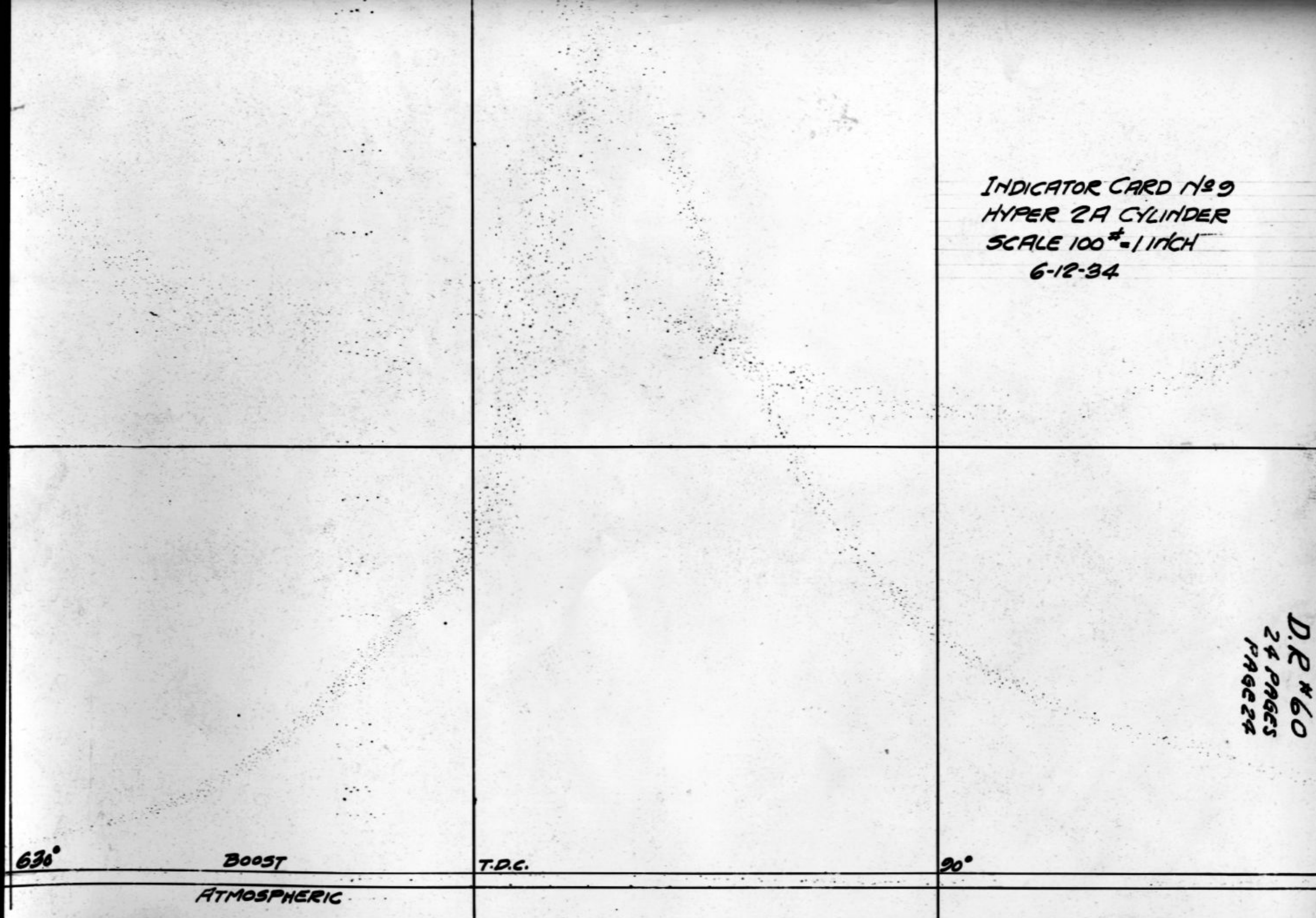
4. "Torsional Vibration of Aircraft Engine Crankshafts" by W. G. Lundquist--Aeronautical Engineering Vol. 5, No. 3.

5. "Torsional Resonance Characteristics of a Twelve Cylinder Vee Aero Engine", British A.R.C. Report #1304 by B.C. Carter and N.S. Muir.

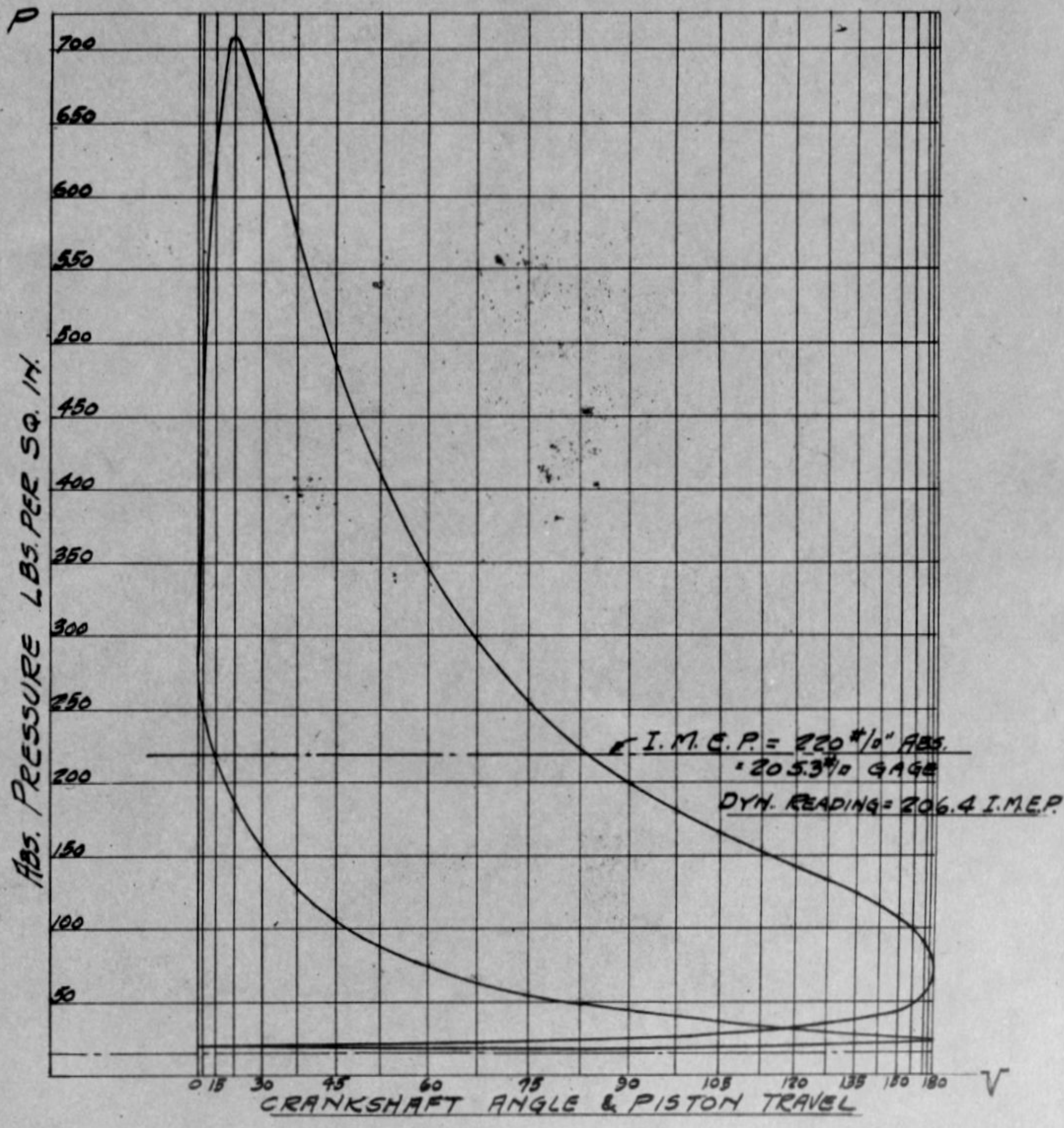
6. "The Geared Marine Oil Engine" by C.H. Bradbury, British Institution of Mechanical Engineers, Proceedings 1933, Vol. 125.

7. "Graphical & Mechanical Computation, Part Two"--Experimental Data, by Joseph Lipke. John Wiley & Sons, Inc.

8. Continental 0-1430-1 Engine Design Report No. 58 "Fundamental Frequency of Torsional Vibration of the Crankshaft and Reduction Gear System".


INDICATOR CARD No 9
HYPER 2A CYLINDER
SCALE 100# = 1 INCH
6-12-34
630°
BOOST
T.D.C.
90°
ATMOSPHERIC
D.R #60
24 PAGES
PAGE 24

C-26

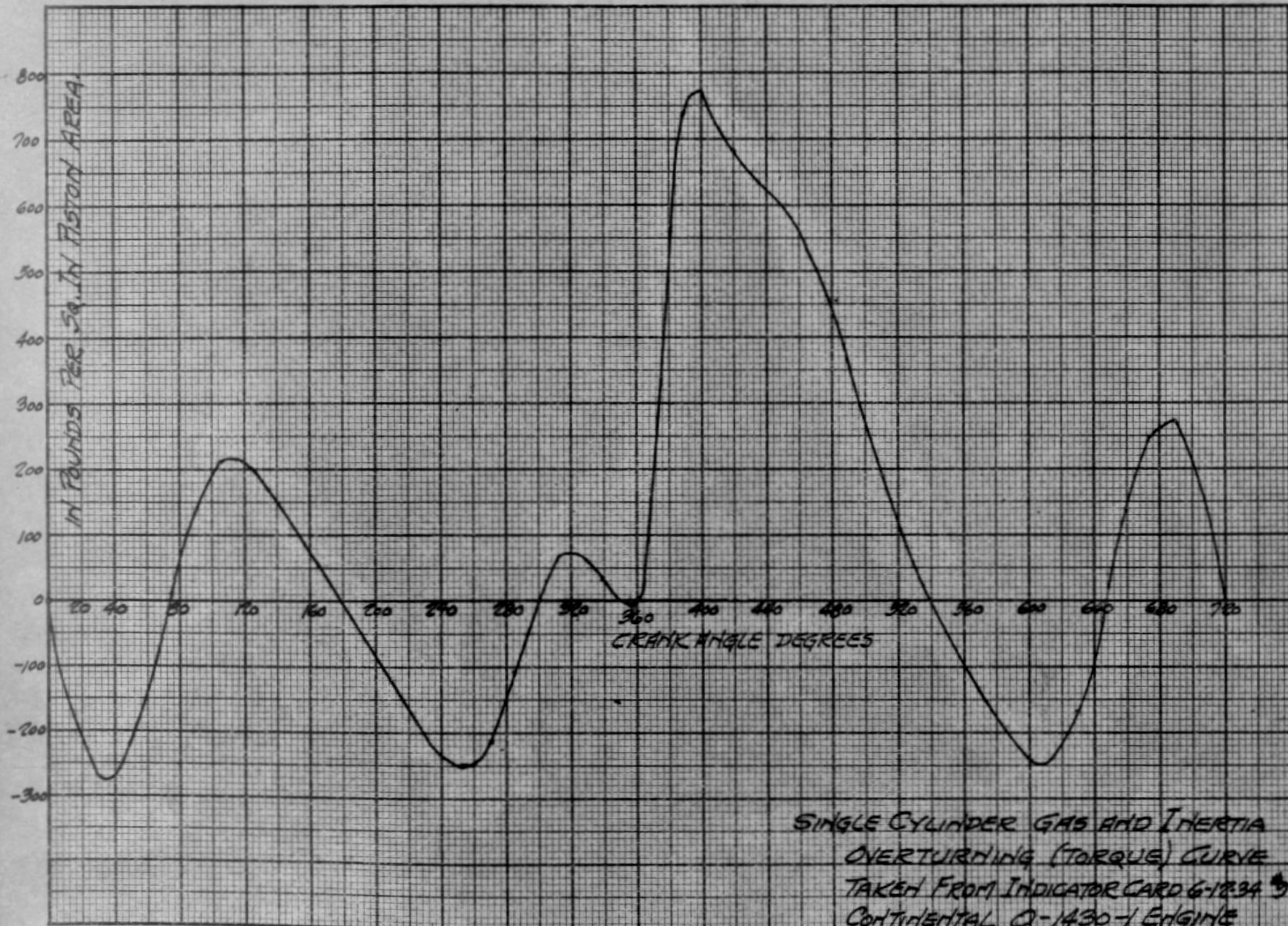


INDICATOR DIAGRAM 6-12-34 No 9.

TAKEN ON FARNBORO INDICATOR HYPER 2A CYL.

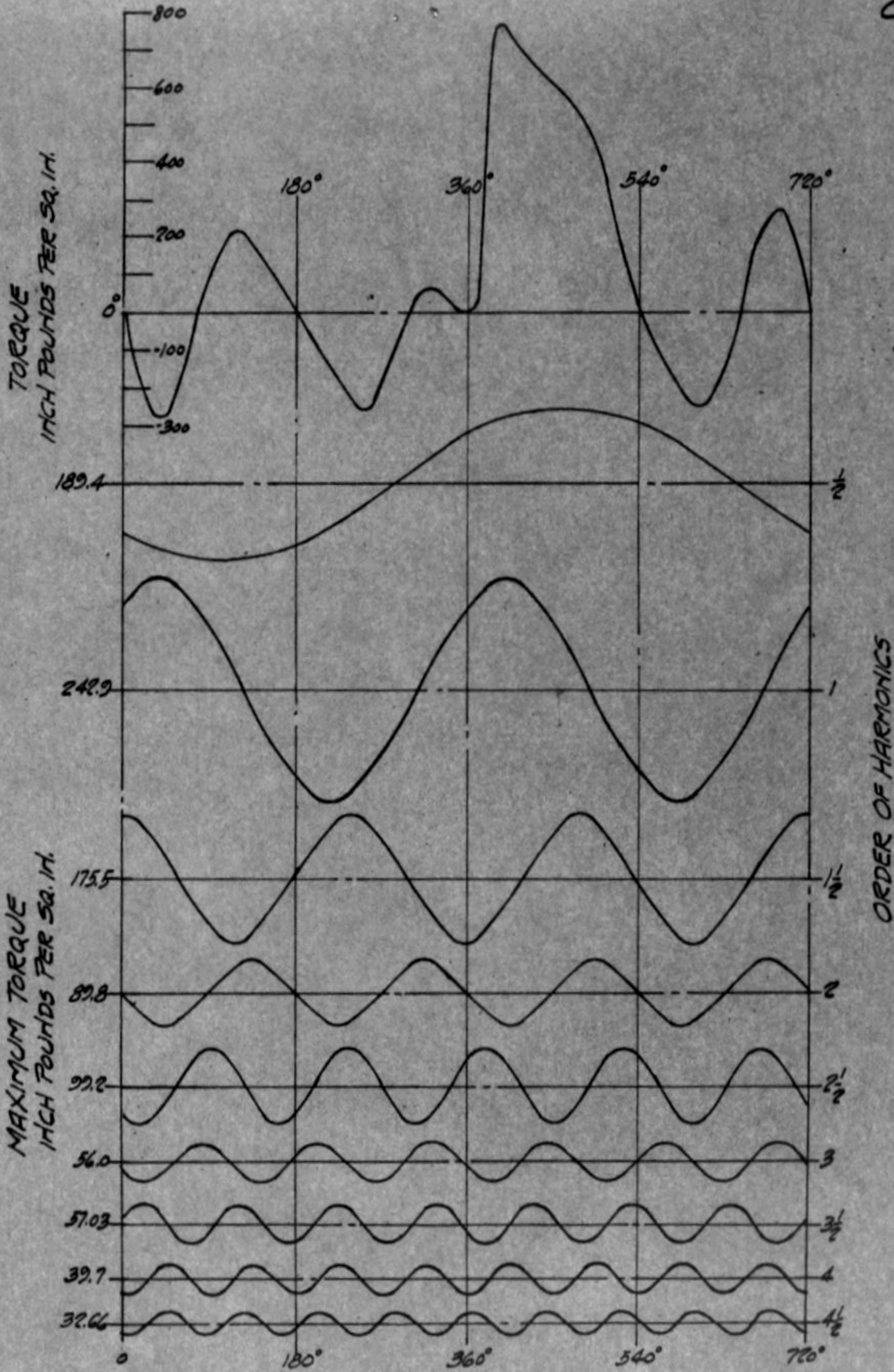
206.4 I. M. E. P. 3071 R. P. M. 18 IN HG. BOOST.

FOR CONTINENTAL O-1430-1 ENGINE

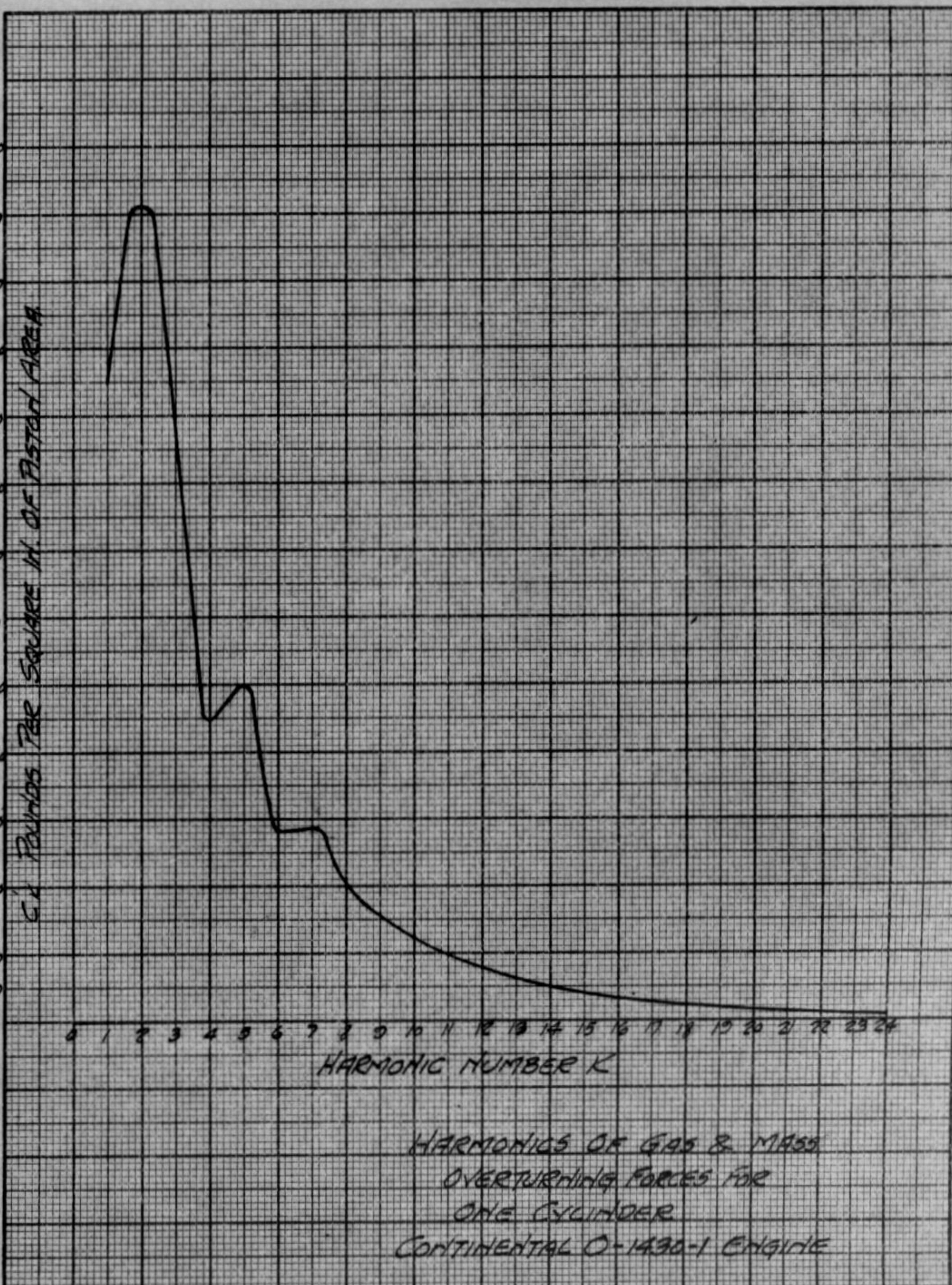


SINGLE CYLINDER GAS AND INERTIA
OVERTURNING (TORQUE) CURVE
TAKEN FROM INDICATOR CARD 6-18-34
CONTINENTAL O-1430-1 ENGINE

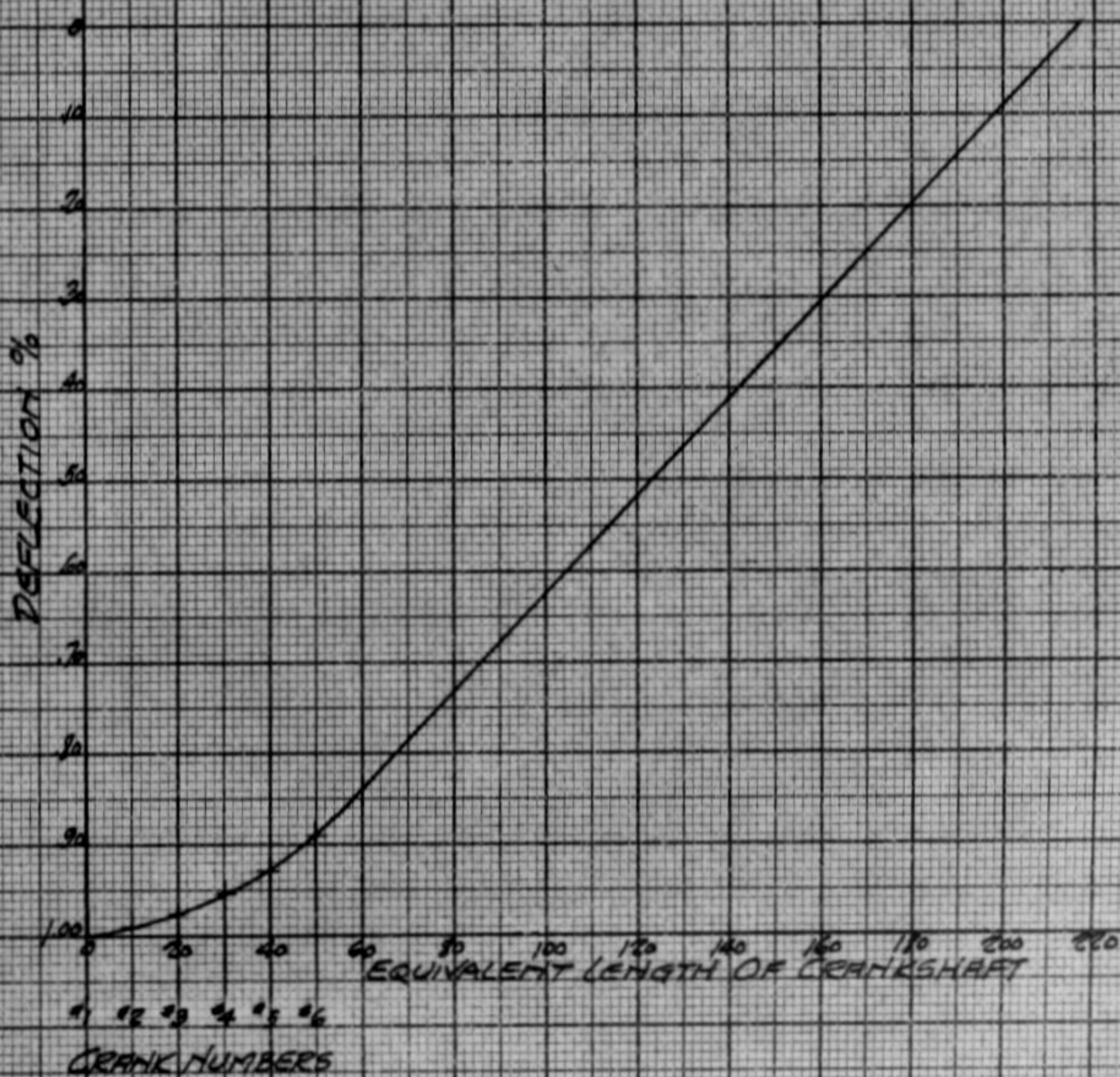
C-27



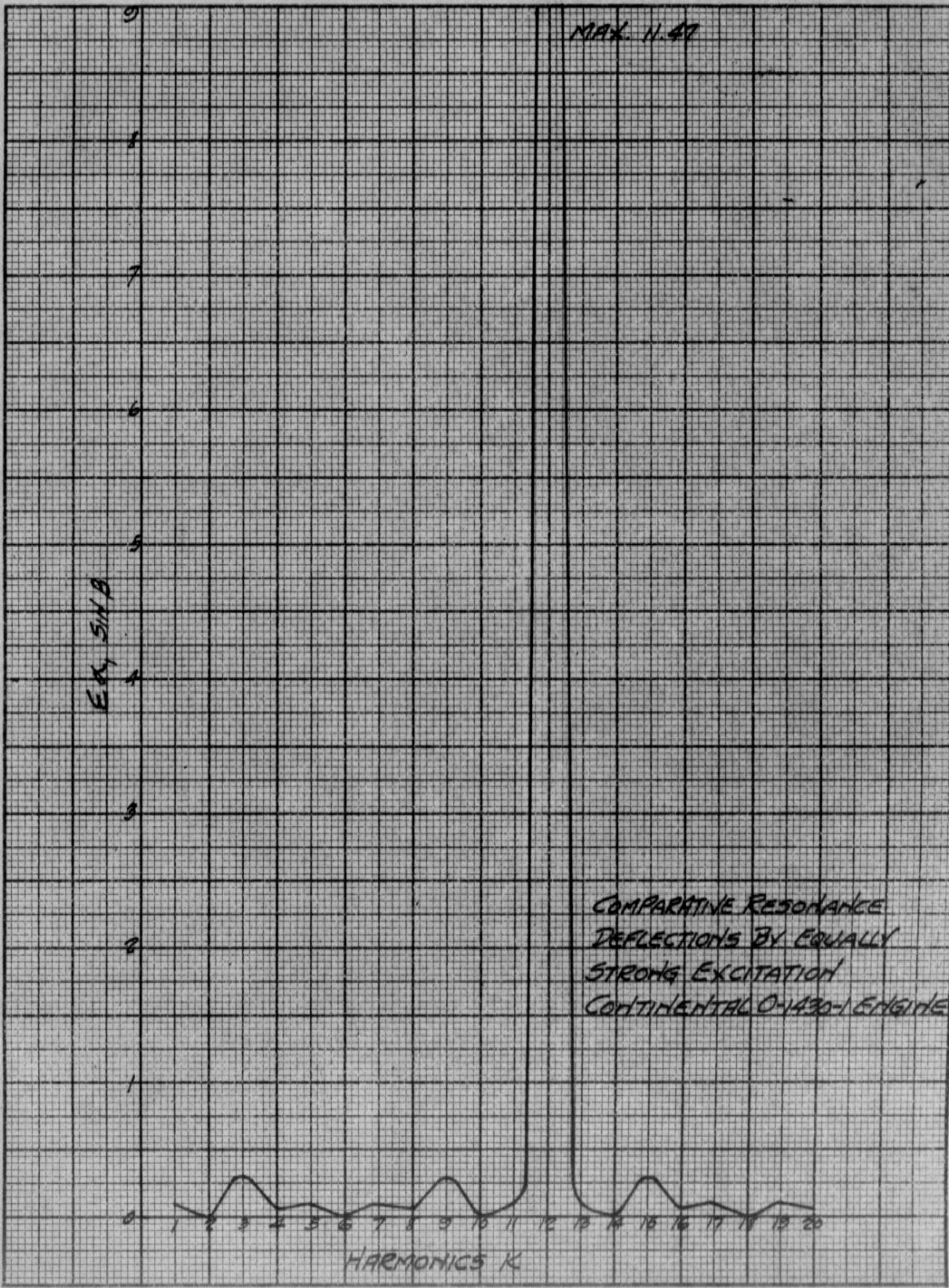
HARMONIC ANALYSIS OF
TORQUE CURVE
CONTINENTAL 0-1430-1 ENGINE.

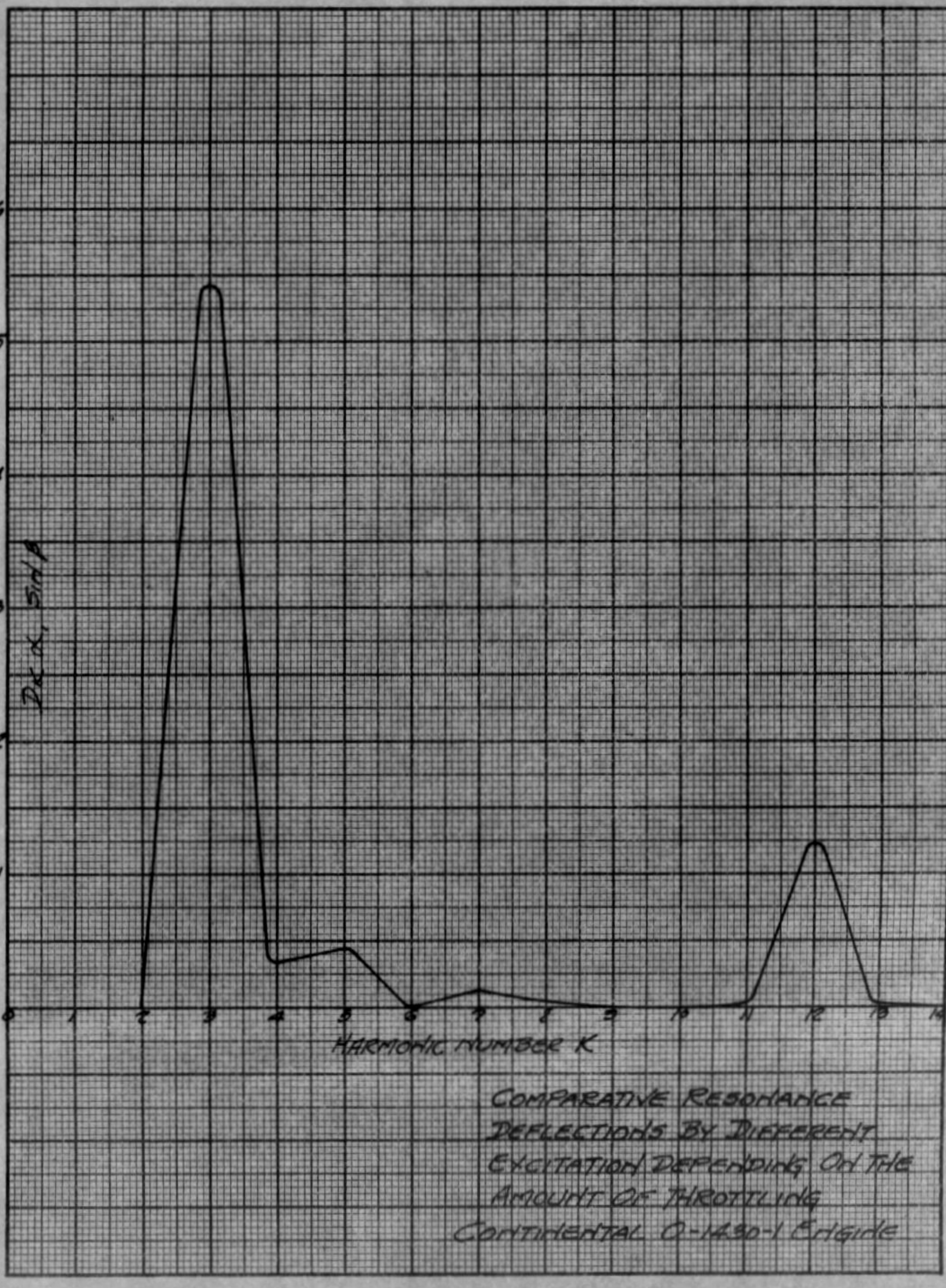


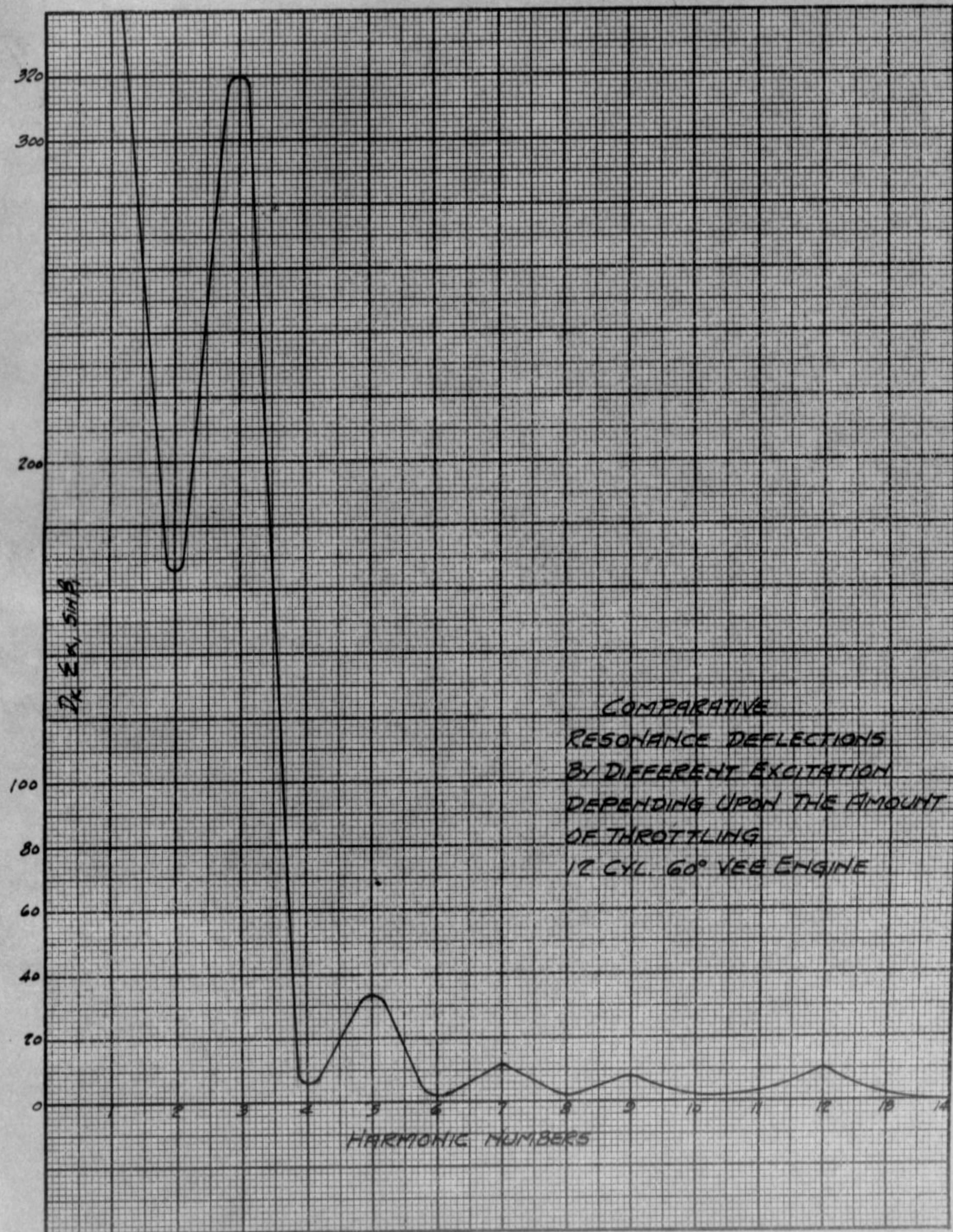
HARMONICS OF GAS & MASS OVERTURNING FORCES FOR ONE CYLINDER CONTINENTAL O-1430-1 ENGINE



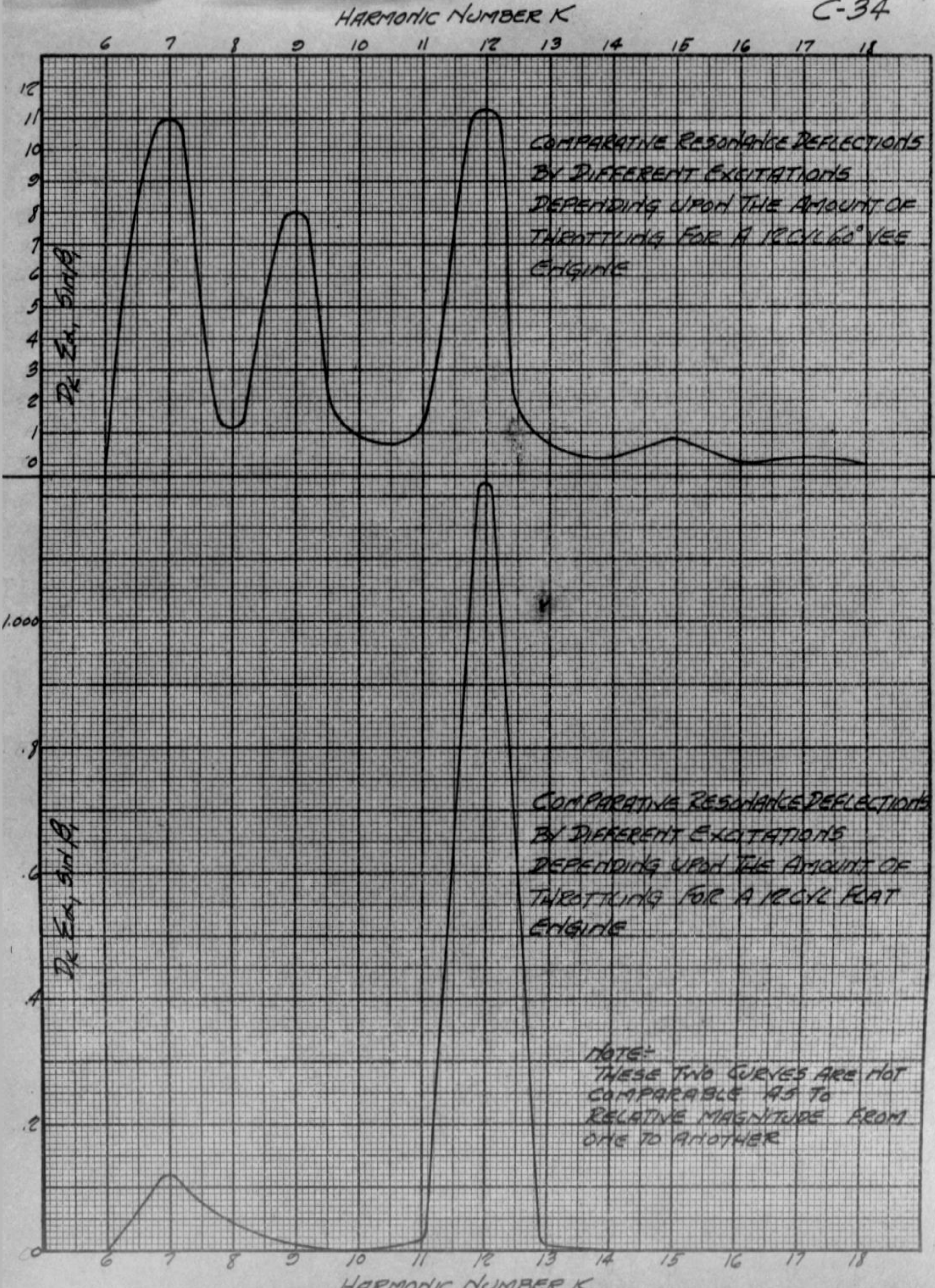
STATIC DEFLECTION CURVE
OF CRANKSHAFT SYSTEM
CONTINENTAL O-1430-1 ENGINE


MAX. N.47
COMPARATIVE RESONANCE
DEFLECTIONS BY EQUALLY
STRONG EXCITATION
CONTINENTAL O-1430-1 ENGINE
$E\alpha_1 \sin\beta$
0
1
2
3
4
5
6
7
8
9
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
HARMONICS K


6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
HARMONIC NUMBER K
COMPARATIVE RESONANCE
DEFLECTIONS BY DIFFERENT
EXCITATION DEPENDING ON THE
AMOUNT OF THROTTLING
CONTINENTAL O-1430-1 ENGINE


320
300
200
100
80
60
40
20
0
Dk Σk, SHB
1
2
3
4
5
6
7
8
9
10
11
12
13
14
HARMONIC NUMBERS
COMPARATIVE
RESONANCE DEFLECTIONS
BY DIFFERENT EXCITATION
DEPENDING UPON THE AMOUNT
OF THROTTLING
12 CYL. 60° VEE ENGINE

C-34


HARMONIC NUMBER K
COMPARATIVE RESONANCE DEFLECTIONS BY DIFFERENT EXCITATIONS DEPENDING UPON THE AMOUNT OF THROTTLING FOR A 12 CYL 60° VEE ENGINE
COMPARATIVE RESONANCE DEFLECTIONS BY DIFFERENT EXCITATIONS DEPENDING UPON THE AMOUNT OF THROTTLING FOR A 12 CYL FLAT ENGINE
NOTE:-
THESE TWO CURVES ARE NOT COMPARABLE AS TO RELATIVE MAGNITUDE FROM ONE TO ANOTHER
HARMONIC NUMBER K