CONTINENTAL AIRCRAFT ENGINE COMPANY

DETROIT, MICHIGAN.

Design Report No. 58

Date April 16, 1934

Sheets SIX

# CONTINENTAL 0-1430-1 ENGINE

## FUNDAMENTAL FREQUENCY OF TORSIONAL VIBRATION OF THE CRANKSHAFT AND REDUCTION GEAR SYSTEM.

PREPARED BY

James W. Kinnucan
James W. Kinnucan
Design Engineer

APPROVED BY

N. N. Tilley
N. N. Tilley
Chief Engineer

# FUNDAMENTAL FREQUENCY OF TORSIONAL VIBRATION OF THE CRANKSHAFT AND REDUCTION GEAR SYSTEM.

References: "Vibration Characteristics of Aircraft Engine Crankshafts" A.C.I.C. #664 by Ford Prescott.
"Vibration Problems in Engineering" by S. Timoshenko.

In the following calculations the method described by Professor S. Timoshenko in his book, "Vibration Problems in Engineering", has been followed. All cranks are reduced to a length of shaft having one torsional rigidity, and the inertia masses reduced to an equivalent flywheel system. The moment of inertia of the propeller is considered as infinite; that is assuming a node at the edge of the propeller hub.

## CALCULATIONS

See sheet #6 for sketch of crankshaft system under consideration.

| | | |
|---|---|---|
| $a$ | = Actual length of crankpin = | 2.500 in. |
| $h$ | = Thickness of crank cheek = | 1.105 in. |
| $a_1$ | = Effective length of crankpin = $(a + 0.9h)$ = | 3.49 in. |
| $2b$ | = Actual length of main journal = | 2.437 in. |
| $2b_1$ | = Effective length of main journal = $(2b + 0.9h)$ = | 3.427 in. |
| $D_1$ | = Outside diameter of main journal = | 3.125 in. |
| $d_1$ | = Inside diameter of main journal = | 2.250 in. |
| $I_1$ | = Moment of inertia of main journal section = $\frac{\pi}{64}(D_1^4 - d_1^4)$ = | 3.42 $in.^4$ |
| $D_2$ | = Outside diameter of crankpin = | 2.875 in. |
| $d_2$ | = Inside diameter of crankpin = | 2.00 in. |
| $q$ | = Offset of inside diameter of crankpin = | .125 in. |

$I_2$ = Moment of inertia of crankpin section = $\frac{\pi}{64} D_2^4 - \left(\frac{1}{64}\pi d_2^4 + \frac{\pi d_2^2}{2} q^2\right)$

| | | |
|---|---|---|
| $I_2$ | = 3.34 - (.785 + .049) = | 2.506 $in.^4$ |
| $F_2$ | = Cross sectional area of crankpin $\frac{\pi}{4}(D_2^2 - d_2^2)$ = | 3.35 $in.^2$ |
| $W$ | = Width of crank cheek = | 3.894 in. |
| $F_3$ | = Cross sectional area of crank cheek = $hw$ = | 4.30 $in.^2$ |
| $I_3$ | = Moment of inertia of crank cheek section = $\frac{hw^3}{12}$ = | 5.40 $in.^4$ |

$E$ = Young's modulus of elasticity for steel = $30 \times 10^6$ $lb/in^2$

$B_2$ = Flexural rigidity of crankpin = $I_2E = 1.474 \times 10^6 (D_2^4 - d_2^4)$

$= 1.474 \times 10^6 (2.875^4 - 2.00^4) = 1{,}474{,}000 \times 52.47 = 77{,}200{,}000$

$B_3$ = Flexural rigidity of crank cheek = $I_3E = 2.5 \times 10^6 hw^3$

$= 2.5 \times 10^6 \times 1.105 \times 3.894^3 = 2{,}500{,}000 \times 65.1 = 162{,}500{,}000$

$C_1$ = Torsional rigidity of main journal = $J_1G = 1.178 \times 10^6 (D_1^4 - d_1^4)$

$= 1.178 \times 10^6 (3.125^4 - 2.250^4) = 1{,}178{,}000 \times 69.63 = 82{,}100{,}000$

$C_2$ = Torsional rigidity of crankpin = $J_2G = 1.178 \times 10^6 (D_2^4 - d_2^4)$

$= 1.178 \times 10^6 (2.875^4 - 2.000^4) = 1{,}178{,}000 \times 52.47 = 61{,}800{,}000$

$C_3$ = Torsional rigidity of crank cheek = $J_3G = 3.33 \times 10^6 \frac{w^3h^3}{(w^2+h^2)}$

$= 3.33 \times 10^6 \times \frac{3.894^3 \times 1.105^3}{(3.894)^2 + (1.105)^2} = 16{,}000{,}000$

$R$ = Crank radius inches = 2.500 in.

K = Factor of complete constraint by main journals (Timoshenko)

$$K = \frac{\frac{R(a+h)^2}{4C_3} + \frac{aR^2}{2C_2} + \frac{a^3}{24B_2} + \frac{R^3}{3B_3} + \frac{1.2}{12 \times 10^6}\left(\frac{a}{2F_2} + \frac{R}{F_3}\right)}{\frac{aR}{2C_2} + \frac{R^2}{2B_3}}$$

$$K = \frac{\left[\frac{2.50(2.50+1.105)^2}{4 \times 16,000,000} + \frac{2.50 \times \overline{2.50}^2}{2 \times 61,600,000} + \frac{\overline{2.50}^3}{24 \times 77,200,000} + \frac{\overline{2.50}^3}{3 \times 162,500,000} + \frac{1.2}{12 \times 10^6}\left(\frac{2.5}{2 \times 3.35} + \frac{2.5}{4.30}\right)\right]}{\frac{2.50 \times 2.50}{2 \times 61,600,000} + \frac{\overline{2.50}^2}{2 \times 162,500,000}}$$

$$K = \frac{.000000509 + .0000001265 + .00000000845 + .0000000321 + .0000000955}{.0000000506 + .0000000192}$$

$$K = \frac{.00000077155}{.0000000698} = 11.05 \text{ inches}$$

$L_1$ = Length of connecting rod = 7.75 in.

$S_1$ = Speed ratio of crankshaft to propeller shaft = 2

$S_2$ = Speed ratio of crankshaft to jackshaft = 1.5

$\ell_1$ = Equivalent length of propeller shaft to first gear =

$$S_1^2\left[\frac{(D_1^4 - d_1^4)}{(D_3^4 - d_3^4)}\right] \times \text{actual length} = 4 \times \left[\frac{69.63}{135.78} \times 3.937 + \frac{69.63}{160} \times 12.125\right]$$

$= 4(2.03 + 5.27) = 29.2$ inches equivalent length.

$\ell_4$ = Equivalent length of jackshaft = $S_2^2\left[\frac{(D_1^4 - d_1^4)}{(D_4^4 - d_4^4)}\right] \times$ actual length

$= (1.5)^2 \times \frac{69.63}{55.53} \times 1.750 = 4.95$ inches equivalent length

$\ell_5$ = Equivalent length of quill shaft = $\frac{(D_1^4 - d_1^4)}{(D_5^4 - d_5^4)} \times$ actual length

$= \frac{69.63}{16.24} \times 10.250 = 43.8$ inches equivalent length

$\ell_3$ = Equivalent length of shaft, center to center of main journals constrained by main bearings =

$$2b_1 + a_1\frac{C_1}{C_2}\left(1 - \frac{R}{K}\right) + 2R\frac{C_1}{B_3}\left(1 - \frac{R}{2K}\right)$$

$$= 3.427 + 3.49\,\frac{82.1 \times 10^6}{61.6 \times 10^6}\left(1 - \frac{2.5}{11.05}\right) + 2 \times 2.50\,\frac{82.1 \times 10^6}{162.5 \times 10^6}\left(1 - \frac{2.5}{2 \times 11.05}\right)$$

$= 3.427 + 3.60 + 2.24 = 9.267$

n = Number of crank throws = 6

Lc = Equivalent length of crankshaft = $l_3 \times \frac{n}{2}$

$= 9.267 \times \frac{6}{2} = 27.801$ inches.

$\frac{R}{L} = \frac{2.5}{7.75}$ $\frac{1}{2}(1 + \frac{R^2}{4L^2}) = .513$

$m_1$ = Rotating weight per crankpin = 6.699 lbs.

$m_2$ = Reciprocating weight per crankpin = 12.767 lbs.

$i_c$ = Calculated moment of inertia of one crank, without counterweights, about the main journal center = 38.21 lbs.in.$^2$

$i_{cwt}$ =Calculated moment of inertia of one counterweight about the main journal center = 22.511 lbs.in.$^2$

I = Moment of inertia of engine system =

$n\left[(m_1 + .513\ m_2)R^2 + i_c\right] + 4i_{cwt} = 6\left[(6.699 + .513 \times 12.767)6.25 + 38.21\right] + 4 \times 22.511 = 817$ lbs.in.$^2$

For complete constraint $f_c = 3.125\sqrt{\frac{C_1}{IL_c}}$

$f_c = 3.125\sqrt{\frac{82,100,00}{27.8 \times 817}} = 3.125\sqrt{3620} = 3.125 \times 60.2 = 188$ vib. per sec.

The $f_c$ above is without consideration of the reduction gear train. From experimental data on geared engines, we find that by the addition of the gear train, the fundamental frequency is cut in half. This data is taken on single stage reduction gears and gears of the "Farman" type. While we have a larger degree of looseness in a two stage reduction gear system, it is believed that but slight error will result in applying the above to this case.

$l_6$ = Equivalent length of crankshaft and gear train, whose frequency

$= \frac{f_c}{2}$ or 94 vib. per sec.

$94 = 3.125\sqrt{\frac{C_1}{Il_6}} = 94 = 3.125\sqrt{\frac{82,100,000}{817 l_6}}$

$8836 = 9.76 \times \frac{82,100,000}{817} \times \frac{1}{l_6}$

$l_6$ = 111 inches equivalent length.

$L_T$ = Total equivalent length of system = 29.2 + 4.95 + 43.8 + 111

$L_T$ = 189 inches

Actual $f$ of the system = $3.125\sqrt{\frac{82,100,000}{189 \times 817}}$

$= 3.125 \times 23.1 = 72.2$ vib. per sec.

Definite knowledge is not at hand as to what harmonics of the torque curve are cumulative for this particular engine. A study of the single cylinder torque curve will be made to determine this.

We will assume, until further knowledge is at hand, the same objectionable harmonics that are found in the 12 cylinder vee type engines.

Critical speeds are located as follows:

$$N = \frac{f(120)}{K}$$

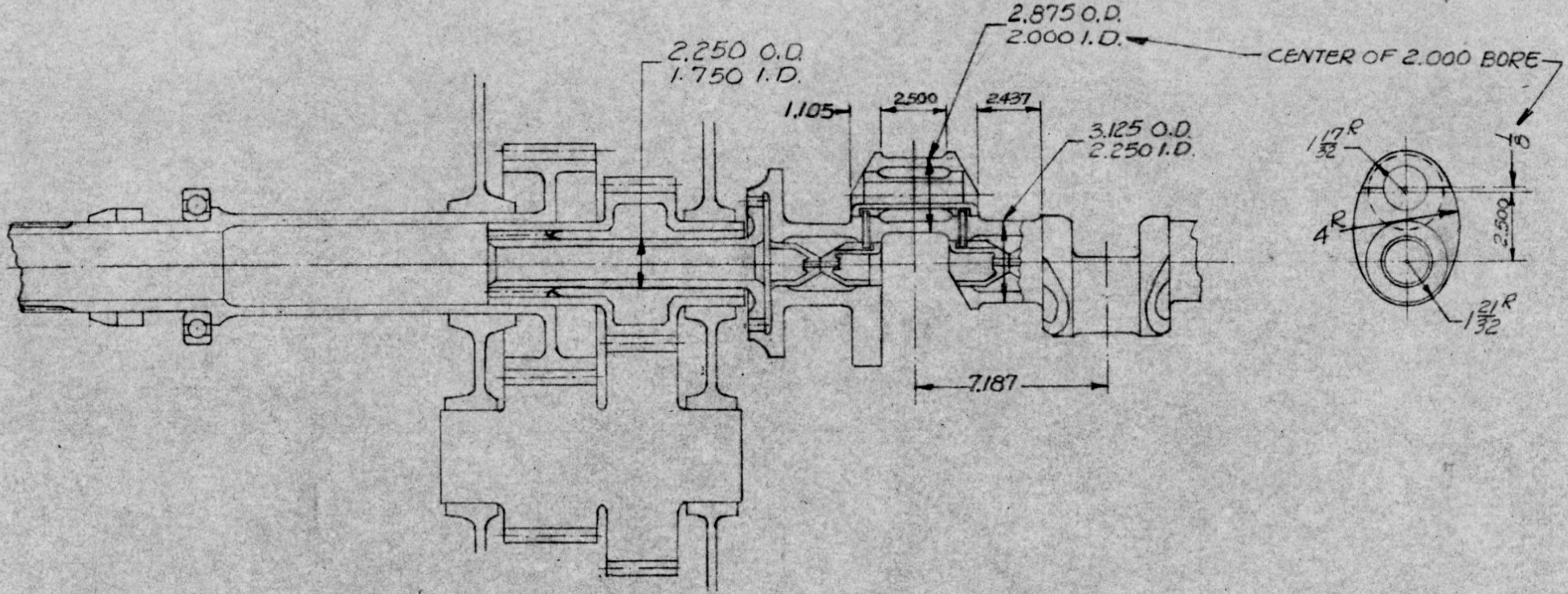
where

- N = Critical speed in R.P.M.
- $f$ = Natural vibration frequency of crankshaft system (cycles per second)
- K = Number of harmonic torque component, known as harmonic coefficient

The known troublesome harmonics of the 12 cylinder vee type are the 12th, 9th, 7th and the 3rd. These harmonics would give critical speeds in the 0-1430-1 engine as follows:

12th harmonic at 722 rpm
9th harmonic at 963 rpm
7th harmonic at 1238 rpm
3rd harmonic at 2888 rpm

From the above it is seen that all the criticals with one exception fall well below the operating range of the engine. A further study is being made to determine the magnitude of this third harmonic, and if it is found to be of sufficient magnitude to be objectionable in the operation of the engine, a change in the length of quill shaft, (which is easily accomplished) might, of necessity, be suggested.




CRANKSHAFT & REDUCTION GEAR SYSTEM

CONTINENTAL O-1430-1 ENGINE

D.R. No 58
SHEET No. 6
6 SHEETS